Angle Pairs - mini edTPA

Task 1

Fall 2013

1. Lesson Background
   ● Teacher’s Name: Kathryn Powers
   ● School: Winton Woods High School
   ● Class: Geometry
   ● Date: Wednesday, September 25, 2013

2. Standards / Objectives
   ● Topic or Essential Question
     ○ In what way can angles be categorized and how can these characteristics and categories aid in solving problems and finding missing angle measures?
   ● Standard(s) Addressed
     ○ CCSS.Math.Content.HSG-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
     ○ CCSS.Math.Content.7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
   ● Instructional Content Objectives
     ○ Successfully list the facts and characteristics of complementary, supplementary, vertical and adjacent angles.
     ○ Successfully construct examples and non-examples of complementary, supplementary, vertical and adjacent angles.
     ○ Successfully apply knowledge of angle pairs and characteristics to solve problems for unknown angles.
   ● Assessment of content objectives
     ○ The students will be given a worksheet consisting of 5 Frayer models that the students will be required to fill out during the class period.
       ■ The first model (Complementary Angles) will be provided as an example by the teachers. The class will be divided into three groups and will be required to fill out the Frayer model designated in their group (Supplementary, Adjacent and Vertical).
       ■ One student from each group will come up to the board to share their Frayer model with the class. During this discussion, the students will check what they have written and make any adjustments necessary and for the definitions that they were not responsible for, students will be recording what their peers are sharing in front of the class.
       ■ The teachers role during this lesson is to act as a guide and help the students if they are struggling by asking questions to get the students to think for themselves.
     ○ Informal in-class questions will measure small-scale student progress and help to bring up misconceptions and misunderstandings.
     ○ Assigned homework will assess student knowledge about applying angle pair characteristics to find missing angle measures, writing equations, and drawing pictures.
3. Language
- Academic Language Functions and Forms (including key lesson vocabulary)

**Congruent angles:** Two angles that have the same measure. The arc marks on the angles below denote congruence.

![Diagram of congruent angles]

**Adjacent angles:** Two coplanar angles with a common side, a common vertex, and no common interior points. The example below shows angles ADB and BDC are complementary angles. Their common side is segment DB, notice that they share a common vertex D and have no common interior points.

![Diagram of adjacent angles]

**Complementary angles:** Two angles whose measures have a sum of 90. The diagram below shows two angles when combined add up to 90 degrees. Although not pictured, complementary angles do not have to be adjacent.

![Diagram of complementary angles]
**Supplementary angles:** Two angles who measures have a sum of 180. The image below shows that angles LMK and KMH are complementary because they add up to 180 degrees, which is a straight line. Notice that angle LMH is a straight line. Although not pictured, supplementary angles do not need to be adjacent.

![Supplementary angles diagram](image)

**Vertical angles:** Two angles whose sides are opposite rays. The vertical (opposite) angles are congruent. The example below shows two sets of vertical angles, 1 and 3, 2 and 4. Notice that the two lines that intersect are straight lines.

![Vertical angles diagram](image)

**Linear pair:** A pair of adjacent angles whose non-common sides are opposite rays. The angles of a linear pair form a straight line. The example below shows that angles ABC and CBD are a linear pair. Angle ABD is a straight line $= 180$ degrees.

![Linear pair diagram](image)

**Linear pair postulate:** If two angles form a linear pair, then they are supplementary.
**Angle bisector:** A ray that divides an angle into two congruent angles and whose endpoint is at the angle vertex.

Segment KM is a bisector of angle LKJ. Note that Segment KM divides angle LKJ into two congruent angles, as denoted by the arc marks. Angle LKM is congruent to angle MKJ

- **Language Objectives**
  - Explain the various angle pair terms with concrete written, visual, and verbal examples.
  - Understand how to solve for missing angle measures using knowledge of angle characteristics and given information.
  - Understand non-examples of specific angle pairs and explain why they are non-examples.
  - Explain how supplementary angles and the linear pair postulate are connected.
  - Visualize and represent word problems in picture form, if needed, to aid the solution process.

- **Assessment of language objectives**
  - Students will fill out angle pair Frayer models with appropriate characteristics, examples, and non-examples for each term.
    - This allows students to find multiple ways to represent a concept/term to foster and enhance understanding.
    - In order to successfully fill out the model, students will have to understand what the term means and how to represent it pictorially and with written word.
  - Students will complete a homework assignment involving angle pair problems, which relies heavily on knowledge and understanding of the key terms.

4. **Differentiation** (including accommodations and modifications of content, materials, delivery, activity, assignment, assessment, etc.)

- **Content:** The college prep (CP) geometry class is expected to know, understand, and apply the terms, but constructions are not required/taught, unlike the honors class. The content presented used basic examples that were familiar to the students, since they had seen many of the examples in the previous lesson.
- **Materials:** The students were provided a packet of Frayer models - somewhat of a template for notes. This hopefully increases student motivation for taking notes, rather than expecting them to start with a blank sheet of notebook paper.
- **Delivery:** A SmartBoard was used to present information - both written and pictorially. Verbal instructions and explanations were also given simultaneously and students could follow along in their packet. This method of delivery addressed the multiple preferred learning styles. The students were learning from their peers in a majority of this lesson. The class could see and hear different ideas and examples from their peers rather than hearing it only from the teacher.
- **Activity:** The activity (having students fill out the Frayer models) allows opportunities for individual and group work, discussion, in-class questions, and creativity in answers/explanation.
- **Assignment/assessment:** The homework assignment, a worksheet printout from the Pearson Common Core online textbook, ensures that the content assessed is aligned with both the information addressed in the class as well as
the Common Core standards. Some visuals were provided on the worksheet (attached below). The students were shown an example of one of the more difficult problems to prevent them from getting frustrated at the difficulty of the problems.

5. Lesson Sequence (Including opening hook and closure - include estimated time allotted next to each phase of the lesson)

A. Opening:
The class begins by pulling up the exploring angle pairs PowerPoint (attached as a PDF file) and handing out worksheets to the students with Frayer models on supplementary, complementary, adjacent and vertical angles. Students are getting out materials and notes for the day while beginning to work on the daily warm-up (which was not discussed during this lesson, but the following day).

B. Discussion / Activity (30 minutes):
The discussion / activity begins as the teacher begins to go through the PowerPoint - discussing the example of the Frayer model of complementary angles. The teacher leads the students in how the Frayer model is set up and how it is useful for the students' conceptual understanding of the mathematical language presented. The teacher explains the definition for complementary angles and how to create/determine the facts and characteristics. The teacher relates these facts/characteristics to a sort of checklist or guideline for the angle pairs. An explanation of the importance of creating visuals for examples as well as non examples then follows. The teacher then shows the class examples and non-examples of complementary angles and asks the students to explain what they are seeing and to explain why or why they are not examples. The students are encouraged to use mathematical language in their explanation of what they are seeing.

Students are then split into three groups (by rows of desks) and each group is given a specific Frayer model to complete. Students are told that they can work together and to use their notes from the prior day. By giving students the opportunity to work alone or discuss with neighbors, the teacher is addressing various preferred learning styles. The students are encouraged to use appropriate mathematical language as they write and discuss the items in the model. The teacher walks around the classroom while the activity is taking place, scaffolding students and providing assistance and clarifications.

Once students are finished with their chart, one student from each group is asked to come up to the board and write what they have on their sheets and to share that information to the rest of the class. If the student is not comfortable speaking in front of the class, accommodations will be made such as allowing another student to explain the model or having the teacher go through it. Appropriate mathematical language is required of the students.

If the students are struggling with any aspect of the chart, the teacher asks the class to help out where there are issues or misconceptions.

Once each group has had the opportunity to share, the teacher asks the class if there are any unanswered questions or any problems with any of the information on the board, such as misunderstandings. The teachers will then present a few strategies that the students can use to help them to remember the definitions of supplementary and complementary angles.

- “C comes before S in the alphabet, 90 comes before 180 on the number line.
- Corner - complementary, angle box to denote a right angle goes in the corner. Straight line - supplementary.
- Giving someone a compliment, you would say to a pair of angles that form a right angle, “you guys are all right”.

The integration of these tricks and mnemonics will hopefully aid those who have trouble remembering definitions and differentiating between similar sounding terms.

After the questions, if there are any, are cleared up, the teachers return to the PowerPoint to introduce the Linear Pair...
Postulate. While this is something new to the students, the teacher will tie in the previous lesson’s information to show that the Linear Pair Postulate is directly related to the definition of supplementary angles.

The last concept of the lesson is then introduced - angle bisectors. The teacher questions the class to gauge their prior knowledge and see if anyone has heard of or knows about angle bisectors. The teacher then goes on to describe what an angle bisector is, what it looks like, and how it can aid in solving problems. In the students’ packets, a Frayer model for angle bisectors is provided so students can fill out the characteristics, examples, and non-examples as the teacher goes over that information in the PowerPoint.

Once the students have an understanding of the linear pair postulate and angle bisectors, the teachers then ask the students to use their understandings of the types of angles presented to determine how to find unknown angle measures in a given problem. Practice problems are put up on the board for the students to discuss.

C. Wrap Up: Homework. Students are given a worksheet to complete. The worksheet is designed to assess the students understanding of complementary, supplementary, adjacent, vertical and congruent angles. The worksheet is also designed to assess the students prior knowledge (in algebra) of setting up and solving equations.

Appendix: Student Worksheet (Homework)
1-5  
Exploring Angle Pairs

Use the diagram at the right. Is each statement true? Explain.
1. $\angle 5$ and $\angle 4$ are supplementary angles.
2. $\angle 6$ and $\angle 5$ are adjacent angles.
3. $\angle 1$ and $\angle 2$ are a linear pair.

Name an angle or angles in the diagram described by each of the following:
4. a pair of vertical angles
5. supplementary to $\angle RPS$
   
   To start, remember that supplementary angles are two angles whose measures have a sum of $\square$.
6. a pair of complementary angles
   
   To start, remember that complementary angles are two angles whose measures have a sum of $\square$.
7. adjacent to $\angle TPU$

For Exercises 8–11, can you make each conclusion from the information in the diagram? Explain.
8. $\angle CEG \cong \angle FED$
9. $\overline{DE} \cong \overline{EF}$
10. $\angle BCE \cong \angle BAD$
11. $\angle ADB$ and $\angle FDE$ are vertical angles.

Use the diagram at the right for Exercises 12 and 13.
12. Name two pairs of angles that form a linear pair.
13. Name two pairs of angles that are complementary.
14. Algebra In the diagram, \(XY\) bisects \(WXZ\).
   a. Solve for \(x\) and find \(m\angle WXY\).
   b. Find \(m\angle YXZ\).
   c. Find \(m\angle WXZ\).

Algebra \(QH\) bisects \(PQS\). Solve for \(x\) and find \(m\angle PQS\).
15. \(m\angle PQR = 3x, m\angle PQS = 4x + 9\)
16. \(m\angle PQS = 4x + 6, m\angle PQR = x + 11\)
17. \(m\angle PQR = 5x - 4, m\angle SQR = 3x + 10\)
18. \(m\angle PQR = 8x + 1, m\angle SQR = 6x + 7\)

Algebra Find the measure of each angle in the angle pair described.
19. The measure of one angle is 5 times the measure of its complement.
20. The measure of an angle is 30 less than twice its supplement.

21. Draw a Diagram Make a diagram that matches the following description.
   - \(\angle 1\) is adjacent to \(\angle 2\).
   - \(\angle 2\) and \(\angle 3\) are a linear pair.
   - \(\angle 2\) and \(\angle 4\) are vertical angles.
   - \(\angle 4\) and \(\angle 5\) are complementary.

In the diagram at the right, \(m\angle HKI = 48\). Find each of the following.
22. \(m\angle HKJ\)
23. \(m\angle IKJ\)
24. \(m\angle FKG\)
25. \(m\angle FKH\)
26. \(m\angle FKJ\)
27. \(m\angle GKI\)
Appendix A: Context for Learning Information

Directions: Use the Context for Learning Information to supply information about your school/classroom context.

I. About the School Where You Are Teaching

1. In what type of school do you teach?
   a. Middle school:
   b. High school
   c. Other (please describe):

2. In what type of community is the school located?
   a. Urban
   b. Suburban (with an urban feel)
   c. Rural
   d. Other (please describe):

3. List any special features of your school or classroom setting (e.g., charter, co-teaching, themed magnet, remedial course, honors course) that will affect your teaching in this learning segment.
   a. Co-teaching/intervention specialist present in the room due to large number of students with IEP’s. Not only does this specialist act as another authority figure, but is better able to assist students struggling with certain exceptionalities. This specialist spends the class visiting with the students with IEP’s to assist them with any issues that they are having as well as making sure they stay on task.
   b. College prep course
   c. Many of the students in this Geometry class are also simultaneously in Algebra 1. Unfortunately this means that their Algebra skills are not up to par which impact their understanding of geometry.
4. Describe any district, school, or cooperating teacher requirements or expectations that might affect your planning or delivery of instruction, such as required curricula, pacing plan, use of specific instructional strategies, or standardized tests.
   a. School/teachers are required to follow the Ohio Common Core standards/curriculum and many teachers have a book (whether tangible or online) to guide their lessons, instruction, and assessment. Thus, our lesson had to match up with the appropriate Common Core standards.
   b. School uses Response to Intervention (RTI) as both a placing plan and academic/behavioral disciplinary method.

II. About the Class Featured in This Assessment

1. What is the name of this course? CP (College Prep) Geometry

2. What is the length of the course?
   a. One semester:
   b. One year:
   c. Other (please describe):

3. What is the class schedule (e.g., 50 minutes every day, 90 minutes every other day)?
   The class schedule is 49 minutes Monday, Tuesday, Thursday, and Friday and 40 minutes on Wednesday.

4. Is there any ability grouping or tracking in mathematics? If so, please describe how it affects your class.
   With RTI, teachers are highly encouraged to give pretests for each section to determine a student's prior knowledge and level of understanding in that given topic. Based on the pretest scores, students are divided into at most 3 groups. The instruction and content is uniform throughout each group, but assignments can be modified to appropriately meet the needs of a particular student.

   However, the cooperating teacher had yet to implement the pretest and ability grouping, so it had little to no effect on the classroom and lesson plan, i.e., the instruction and homework were the same for each student.

5. Identify any textbook or instructional program you primarily use for mathematics instruction. If a textbook, please provide the title, publisher, and date of publication.
   1. Geometry by McDougall Littell (students use for homework problems, additional aid)
   2. Geometry Common Core by Pearson electronic version (pearsonsuccessnet.com) (used to guide instruction, create lesson plans, includes additional resources such as worksheets and example problems)

6. List other resources (e.g., electronic white board, graphing calculators, online resources) you use for mathematics instruction in this class.
   a. Smart Board (electronic white board)
   b. Scientific calculators
   c. Online Common Core Geometry textbook
   d. Chalkboard
   e. Overhead projector

III. About the Students in the Class Featured in This Assessment
1. Grade level composition (e.g., all seventh grade; 2 sophomores and 30 juniors):

   1 freshman, 21 sophomores, 3 juniors and 2 seniors

2. Number of students in the class __27___ males __17___ females __10___

3. Complete the chart below to summarize required or needed supports, accommodations or modifications for your students that will affect your instruction in this learning segment. As needed, consult with your cooperating teacher to complete the chart. The first two rows have been completed in italics as examples. Use as many rows as you need.

Consider the variety of learners in your class who may require different strategies/supports or accommodations/modifications to instruction or assessment.

- English language learners
- Gifted students needing greater support or challenge
- Students with Individualized Education Programs (IEPs) or 504 plans Struggling readers
- Underperforming students or those with gaps in academic knowledge

<table>
<thead>
<tr>
<th>Learning Needs Category</th>
<th>Number of Students</th>
<th>Supports, Accommodations, Modifications, and/or Pertinent IEP Goals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example: Struggling readers</td>
<td>5</td>
<td>Provide oral explanations for directions and simplified text for word problems</td>
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<tr>
<td>IEP - Other Health Impairments</td>
<td>2</td>
<td>Intervention specialist in the classroom to assist students with IEP goals.</td>
</tr>
<tr>
<td>IEP - Intellectual Disabilities</td>
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<td>Intervention specialist in the classroom to assist students with IEP goals.</td>
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<tr>
<td>IEP - Specific Learning Disability</td>
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</tr>
<tr>
<td>IEP - Not Identified</td>
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<td>Intervention specialist in the classroom to assist students with IEP goals.</td>
</tr>
<tr>
<td>Title I</td>
<td>2</td>
<td>Intervention specialist in the classroom to assist Title I students.</td>
</tr>
<tr>
<td>504 Plan</td>
<td>2</td>
<td>Intervention specialist in the classroom to assist students under the 504 Plan.</td>
</tr>
<tr>
<td>Limited English Proficient</td>
<td>3</td>
<td>Provide visual examples and</td>
</tr>
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</table>
non-examples. Intervention specialist in the classroom to assist LEP students.

<table>
<thead>
<tr>
<th>Gifted - Visual Performing Arts</th>
<th>1</th>
<th>Students are encouraged to explain / show their understanding in multiple ways, including creating diagrams and drawings.</th>
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</thead>
<tbody>
<tr>
<td>Gifted - Visual Performing Arts and Reading</td>
<td>1</td>
<td>Students are encouraged to explain / show their understanding in multiple ways, including creating diagrams and drawings.</td>
</tr>
<tr>
<td>Gifted - Science</td>
<td>1</td>
<td>Students are encouraged to explain / show their understanding in multiple ways, including creating diagrams and drawings.</td>
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</table>

Appendix B: Planning Commentary

Directions: Respond to the prompts below (no more than 9 single-spaced pages, including prompts).

I. Central Focus

1. Describe the central focus and purpose for the content you will teach in this learning segment.

   The central focus of this lesson is to build upon the prior lesson in order to develop a conceptual understanding of complementary, supplementary, vertical, adjacent angles, and the angle bisector. This lesson is also designed to reinforce the angle addition postulate and the use of algebra by reintroducing variables and problems that require the students to set up equations with variables. The angle pair discussion is a building block for more difficult items that will come up later in Geometry. Having a strong conceptual understanding of the topics in this lesson will help students as they prepare for the next chapter which is Reasoning and Proof.

2. Given the central focus, describe how the standards and learning objectives within your learning segment address

   a. Conceptual understanding
      i. Use of open ended questions to gauge and monitor student understanding as well as address any misconceptions
         1. What does it mean to be a complementary/supplementary/vertical/adjacent angle?
         2. How can you tell that two angles are congruent?
         3. If an angle looks like it’s 90 degrees, does that mean it is?
         4. What would a picture of _____ look like?
      ii. The use of the Frayer models where the students are required to come up with facts and characteristics as well as visual examples and non-examples.
         1. Allows students to represent a term in multiple ways, appeals to multiple kinds of learners, reinforces conceptual understanding rather than rote memorization
b. Procedural fluency
   i. Students must not only understand what a particular angle pair is, but how to use this knowledge to solve math problems (such as finding a missing angle measure).
   ii. By introducing and reviewing the terms and their multiple representations first, students hopefully gain a deeper understanding of concepts.
   iii. Conceptual understanding and procedural fluency should be thought of together. A solid conceptual understanding leads to better retention of procedural fluency and one’s ability to solve problems.
   iv. In their homework assignment, students are asked not only to draw the different types of angle pairs, but to solve problems that are reliant on the conceptual understanding of the angle pairs, angle bisectors, linear pair postulate, and basic algebra.
   v. Providing in class examples gives students the opportunity to see and hear the procedures necessary for a given problem.

c. Mathematical reasoning and/or problem solving skills
   i. Mathematical reasoning and problem solving skills go even further in that students are not just remembering how to solve a problem, but why a certain procedure is used to solve that problem.
   ii. Students are expected to take given information and understand how it relates to the problem at hand.
   iii. A focus on “clues” (visual markings, given characteristics, key words) pushes students to understand what a problem is truly asking and what knowledge they will use to solve it.

3. Explain how your plan builds on lessons that came before to help students make connections between facts, concepts, and procedures, and to develop their reasoning and/or problem solving skills to deepen their learning of mathematics.
   a. The angle pair terminology was actually introduced during the lesson the day before, so our lesson was deeper review as well as an introduction to the linear pair postulate and angle bisector. This lesson is designed to identify any misconceptions and to build upon the prior lesson to strengthen their conceptual understanding.
   b. The idea and procedure of adding angle measures draws on the concept from the previous section, segment addition.
      i. The same ideas are involved, including creating expressions and setting up equations to solve for certain measures.
   c. Algebra (more specifically, solving algebraic expressions) is heavily used in this class. Students will be using algebra and angle pair knowledge to solve for missing angle measures. Requiring students to continue to use and practice their algebra skills is a way for them to strengthen their problem solving skills as they will continue to see algebra problems throughout their school career.
II. Knowledge of Students to Inform Teaching

For each of the prompts below (II.1–3), describe what you know about your students with respect to the central focus of the learning segment.

Consider the variety of learners in your class who may require different strategies/support (e.g., students with IEPs, English language learners, struggling readers, underperforming students or those with gaps in academic knowledge, and/or gifted students).

1. Prior academic learning and prerequisite skills related to the central focus—What do students know, what can they do, and what are they learning to do?

   Students have a prior knowledge of complementary, supplementary, adjacent and vertical angles as they were presented for the first time in the previous lesson. The students have been exposed to examples and non-examples, but have not created their own examples of these angle pairs. Most students have a good understanding of these angle pairs, but have yet to master them and use them successfully to solve related problems. In this lesson, students are learning to use their conceptual understanding of angle pairs to solve problems such as explaining and drawing the different angle pairs. Students are also learning new terminology such as the linear pair postulate which builds upon their prior knowledge of supplementary angles.

   The students have experience with the angle addition postulate (which was built upon their prior knowledge of the segment addition postulate). In the homework after this lesson, students will be solving problems that require them to use the angle addition postulate and information from this lesson to find missing information from a diagram. Students are expected to know basic algebra, such as solving equations, but many still struggle with this skill. They are learning to recall and apply this information to geometry concepts that introduce variables into the angle addition postulate.

   This lesson reinforces that it is important to understand the information a diagram can and cannot provide. Students are tempted to make assumptions based on the appearance of a diagram, but assumptions must be limited to special geometric relationships. Students must understand that certain assumptions cannot be made without specific clues/instructions. This lesson builds upon their prior knowledge and helps them to understand previous misconceptions.

2. Personal/cultural/community assets related to the central focus—What do you know about your students’ everyday experiences, cultural backgrounds and practices, and interests?

   a. Many students are on free or reduced lunch, so poverty is a relevant issue for much of the school.
   b. Sports are huge interests - it seems that some students only try in school to stay on their sports teams.
   c. Varied cultural/racial backgrounds: largely African American and Hispanic students, English language learners (Spanish and French speakers)
   d. The Arts are very important in the school. There are two students in the class that are gifted in visual arts.

3. Mathematical dispositions—What do you know about the extent to which your students

   a. perceive mathematics as “sensible, useful, and worthwhile”
      i. Very few, even those students who easily grasped new mathematical knowledge and theories have a positive disposition toward mathematics. Several students in the class do not write notes even when they are prompted to write the information down because that particular information will be on the test. These same students would not copy down examples that were discussed in class even on homework that was to be turned in for a grade.

   b. persist in applying mathematics to solve problems
Again, very few. If a student did not know how to start a problem or even what the problem was asking, he or she would not even try or consult his or her notes. Many students in the class would not try problems unless a teacher stood over them to make sure they were doing their work, especially with the warm up problems that are given each day. Several students in the class would get frustrated with a problem and ask for help, and then get even more frustrated when they were not given the answer, and tend to give up without being pushed.

c. believe in their ability to learn mathematics

As mentioned earlier, students would tend to give up easily and had the belief that the math was too difficult for them to understand or to learn. When students are “tutored” and they figure out the solution, they believe that they can do it, but many students do not feel that they are able to figure out mathematics on their own - scaffolding is necessary to foster motivation.

III. Supporting Students’ Mathematics Learning

Directions: Respond to prompts below (III. 1–3). As needed, refer to the instructional materials and the lesson plan you have included to support your explanations. Use principles from research and/or theory to support your explanations, where appropriate.

1. Explain how your understanding of your students’ prior academic learning, personal/cultural/community assets, and mathematical dispositions (from prompts II. 1–3 above) guided your choice or adaptation of learning tasks and materials.

This lesson was designed based on the theory of Multiple Intelligences by Howard Gardner. The Frayer model lesson we designed took into consideration those students with the following intelligences:

- **Linguistic** - this lesson allows for students to work together and engage dialogue with one another about the information presented to them. Students are then allowed to verbally present their information to the class.
- **Logical-Mathematical** - the lesson and the homework gave students the opportunity to use their inductive and deductive reasoning to come up with examples and non-examples of the different angle pairs.
- **Spatial** - the use of diagrams and visual representations allow spatial learners to recognize patterns and properties of the angle pairs.
- **Interpersonal** - this lesson allows students to work together to discuss ideas and to work effectively to solve a problem.
- **Intrapersonal** - this lesson allowed students that learn better by themselves to construct their knowledge on their own.

Knowing that the students were struggling in the prior days discussion, we realized the importance to give the students another opportunity to go over the material in a different format that would touch on the different learning styles of the class. We decided to use the Frayer model in order to give the class a different way to view the material, as well as getting them to construct their own learning as well as learning from one another. The Frayer model allows for written, verbal and visual responses to the prompts. This format allows for students with different learning types to be engaged with the lesson, especially the ELL students in the classroom. Additionally, those who lack interest in the subject of mathematics might find this lesson to be more interesting or refreshing compared to previous lessons.

Also realizing that many students struggle with the integration of algebra and problem solving into geometry, our focus on that day’s lesson was to reiterate the essential terms and concepts. It was our hope that this would build a foundation and that students would find it easier to transition into problem solving having a solid base knowledge of the relevant terms.

2. Describe and justify why your instructional strategies and planned supports are appropriate for the whole class and
students with similar or specific learning needs.

The students in this class tend to be disengaged during a typical note taking session. This lesson was designed to keep the class engaged by introducing a new note taking technique where students would be creating their own notes. We allowed the students to work with each other in order to foster a cooperative learning environment. This lesson was planned to be beneficial for most learning types as it uses words and visuals that the students construct on their own or in groups.

Since it was observed that some students liked to work alone, while others preferred consulting within peers. The Frayer model activity appealed to both preferences.

Furthermore, because of the presence of English Language Learners in the classroom, as well as those who were gifted in the visual arts, we found it crucial to include visuals. The Frayer model allows students to represent a term in multiple formats, helping to increase conceptual understanding.

3. Describe common mathematical preconceptions, errors, or misunderstandings within your content focus and how you will address them.
   ● Segment addition postulate vs angle addition postulate - no connection. The students did not realize that the same procedures used in solving segment addition problems could be applied to angle addition problems.
     ○ If there is time, we will address this by showing an example problem and how its setup and basic problem solving procedure matches that of segment addition. If the example presented is too difficult for the students, we will present an easier problem for the students to solve on their own.
   ● Arc marks on angles must be in a specific area in order for the angles to be congruent.
     ○ Even if the arc marks are there (indicating congruent angles), if the two angles do not look the same, the students will argue that they are not congruent, despite the markings that tell them otherwise.
     ○ This misconception will be addressed by explanation, prompting, and questioning. Furthermore, the analogy of the paper dollar and silver dollar, which look completely different, but are “congruent” in value will hopefully aid in the understanding.
   ● 180 degrees is not a straight line.
     ○ Direct instruction should fix this misconception. It is simply a matter of making connections and remembering definitions.
   ● Struggling with the idea of variables (“x”) and how they related to the geometric concepts they are learning
     ○ When introducing how to solve for a missing angle measure, we will go step by step and wait to introduce variables until students grasp the idea (i.e. I know these two angles are supplementary, so if this angle is 85 degrees, the other one has to be 95 degrees)

   ● Angles can be named in any order.
     ○ Students have the misconception that you can represent angle LKM as angle MLK. In order to correct this misconception, the teacher will physically trace from L to K to M to show that the vertex must be the middle letter in their angle description. The teacher will show that the only way to connect M to L is to go through the vertex. The teacher will also show that it is acceptable to
represent angle LKM as angle MKL, again tracing the rays with their finger or a pointer, and will encourage the students to do the same as they try to represent the angle(s).

IV. Supporting Mathematics Development Through Language

1. **Language Demand:** Language Function. Identify one language function essential for students to learn the mathematics within your central focus. Listed below are some sample language functions. You may choose one of these or another more appropriate for your learning segment.
   a. Compare/contrast
   b. Conjecture
   c. Describe
   d. Explain
   e. Prove

   This lesson was created as a way for students to demonstrate their knowledge of key terms by explaining in words and diagrams. This lesson also gives students the opportunity to discuss with the class their understandings. We really want to stress the language objectives and vocabulary, as they are a building block to more advanced concepts and problems.

   In addition, students are encouraged to describe (whether verbally, pictorially, or with written word) the various geometric terms and how they can be used to solve a given problem. If a student is able to describe how he or she arrived at an answer using certain mathematical knowledge, there is more evidence of conceptual understanding and problem solving skills.

2. Identify a key learning task from your plan that provides students with opportunities to practice using the language function.

   During the activity portion of the lesson, the students are writing in their own words, the characteristics and facts of supplementary, adjacent and vertical angles, as well as drawing pictures of examples and non-examples. Students are encouraged to work together to share their thoughts and ideas about what each angle pair means to them. They are practicing the functions of description and explanation through various mediums in order to build conceptual knowledge, procedural fluency, and eventually, mathematical problem solving.

Homework:

- Questions 1-3 asks the students if each statement is true and then it tells them to explain.
  - 1. Angle 5 and Angle 4 are supplementary angles.
  - 2. Angle 6 and Angle 5 are adjacent angles.
  - 3. Angle 1 and Angle 2 are a linear pair.
- Questions 4-7 asks the students to name an angle or angles with the following descriptions. Students must understand the mathematical language to solve the problems.

  In questions 1-3, students are provided with a diagram and are asked to identify if the statements are true by
evaluating the information give in the diagram. Students are then asked to explain why they answered true or false - this is where the student practices using the language function.

In questions 4-7, students are asked to name an angle or angles given a description. Students are using the language function to draw upon their knowledge of the definitions / facts / characteristics of the different angle pairs.

3. **Additional Language Demands.** Given the language function and task identified above, describe the following associated language demands (written or oral) students need to understand and/or use.
   a. **Vocabulary and/or symbols**
      Students need to understand the symbols for a right angle and congruent angles. Students also need to be able to understand the terms supplementary, complementary, adjacent and vertical angles. Students also need to understand that the location of the congruent marks do not have to be in a specific location. The students must not make assumptions when dealing with the diagrams of angles based on how they appear to the eye, but they must make use of the clues and/or the information they are given in the problems or examples. They must draw upon their prior knowledge to identify key vocabulary terms and symbols.

   b. **Mathematical precision (e.g., using clear definitions, labeling axes, specifying units of measure, stating meaning of symbols), appropriate to your students’ mathematical and language development**
      When constructing examples and non-examples, students must adhere to the mathematical definitions. We included the facts/characteristics portion of the Frayer model in hopes that students would use it as a guide and checklist while creating these examples and non-examples. Students must be able to recognize acute and obtuse angles and be able to draw and label them correctly. It is also critically important for students to label their angles appropriately and to describe them appropriately. While angle ABC is the same as CBA, angle ABC is not the same as angle BAC. Students must understand when explaining an angle the vertex must be in the middle. Students must be able to recognize and appropriately use the symbols that identify congruent and right angles.

   c. **Plus at least one of the following:**
      i. **Syntax**
         Students must understand that stating that an angle is a right angle can be done by stating that the angle is 90 degrees, or by marking the angle with a corner box. In the same way, many mathematical concepts can be stated in various ways and the more students are able to draw connections between the representations, the better able they are to solve related problems. The markings on the diagram must be used to determine defining characteristics of the diagram. Assumptions should not be made by the "look" of an angle, but prior knowledge of indicators must be drawn upon in order to understand a graph or diagram.

      ii. **Discourse**

4. **Language Supports.** Refer to your lesson plan and instructional materials as needed in your response to the prompt. Describe the instructional supports (during and/or prior to the learning task) that help students understand and successfully use the language function and additional language identified in prompts IV 1–3.
To support students use of vocabulary and symbols, the lesson provides students with the definitions of the various angle pairs, students are then required to complete the handout that includes areas for characteristics/facts, visual examples and visual non-examples. Students are encouraged to come up with their own examples and non-examples. The teachers are scaffolding the students by asking open ended questions related to their confusion or misconceptions.

To support students mathematical precision with angle pairs, the lesson requires students to draw examples and non-examples of angle pairs. The teachers are walking around the classroom during the activity to assist students that are having difficulty. The teachers are making note of common misconceptions and will present those common misconceptions to the class and ask questions as to why they are or why they are not good representations. During this phase of the lesson, the teachers are making sure that students are using appropriate mathematical language as well as drawing precise diagrams.

V. Monitoring Student Learning. Refer to the assessments you will submit as part of the materials for Task 1.

1. Describe how your planned formal and informal assessments will provide direct evidence of students’ conceptual understanding, procedural fluency, and mathematical reasoning and/or problem solving skills throughout the learning segment.

**Conceptual Understanding:** Informal assessment of students conceptual understanding are assessed at the beginning of the lesson when students are asked open ended questions about the different angle pairs. Students are also asked to draw examples and non-examples of the the various angle pairs. Students are also informally assessed when they are asked questions such as:

How do you know that?
Why is that an example?
Why is that a non-example?
What are the clues that tell you that?

The formal assessment is with the homework where students are asked questions that apply their understanding of the various angle pairs and their definitions and characteristics.

**Procedural Fluency:** The purpose of this lesson is to improve the conceptual understanding of angle pairs with the students which will help them with their procedural fluency. In their homework assignment, students are asked not only to draw the different types of angle pairs, but to solve problems that are reliant on the understanding of the angle pairs, angle bisectors, linear pair postulate, and basic algebra.

**Problem Solving:** Mathematical reasoning and problem solving skills go even further in that students are not just remembering how to solve a problem, but why a certain procedure is used to solve that problem. Students are expected to take given information and understand how it relates to the problem at hand. A focus on “clues” (visual markings, given characteristics, key words) pushes students to understand what a problem is truly asking and what knowledge they will use to solve it. The homework assignment assesses the student’s ability to use mathematical reasoning to solve algebraic equations introduced into an angle additions postulate. In questions 14 - 18, students are assessed on their ability to recognize the definition of an angle pair and how that definition will help them solve a problem where they need to create an algebraic equation to solve the problem. In questions 19 and 20, there is less information given and the students will be assessed on their problem solving skills where they are required to set up an equation where they are not given the components of the equation, but must set one up using their knowledge of complementary and supplementary angles.
2. Explain how the design or adaptation of your planned assessments allows students with specific needs to demonstrate their learning.

The lesson was designed to address the various needs of the multiple learning types based on Gardner’s Multiple Intelligences. Information on the in class worksheet includes vocabulary, as well as visual examples and non-examples. Students are given the opportunity to work alone or in a group, to address their preferred learning style. Students will have the opportunity to represent their information verbally (in groups, at the board or by answering open-ended questions presented throughout the lesson) and symbolically (by drawing examples and non-examples of the various angle pairs). Teachers also verbally explain terminology and graphs as well as giving visual examples and non-examples.

ELL students are able to read and hear the verbal explanations as well as see the visual representations of the various angle pairs. These students are able to see not only examples, but non-examples as well and are given visual and verbal explanations as to why they are or are not examples.

In the informal assessment, the students are able to record information that is presented by the teacher or by their peers as well as demonstrate their own knowledge by drawing examples and non-examples.

In the formal assessment, the homework gives the students the verbal and visual representations of the problems. The homework addresses those with special needs by giving them the ability to show multiple representations of their knowledge on the formal assessment by explaining with words, while other questions require them to draw pictures to demonstrate their knowledge of the various angle pairs.